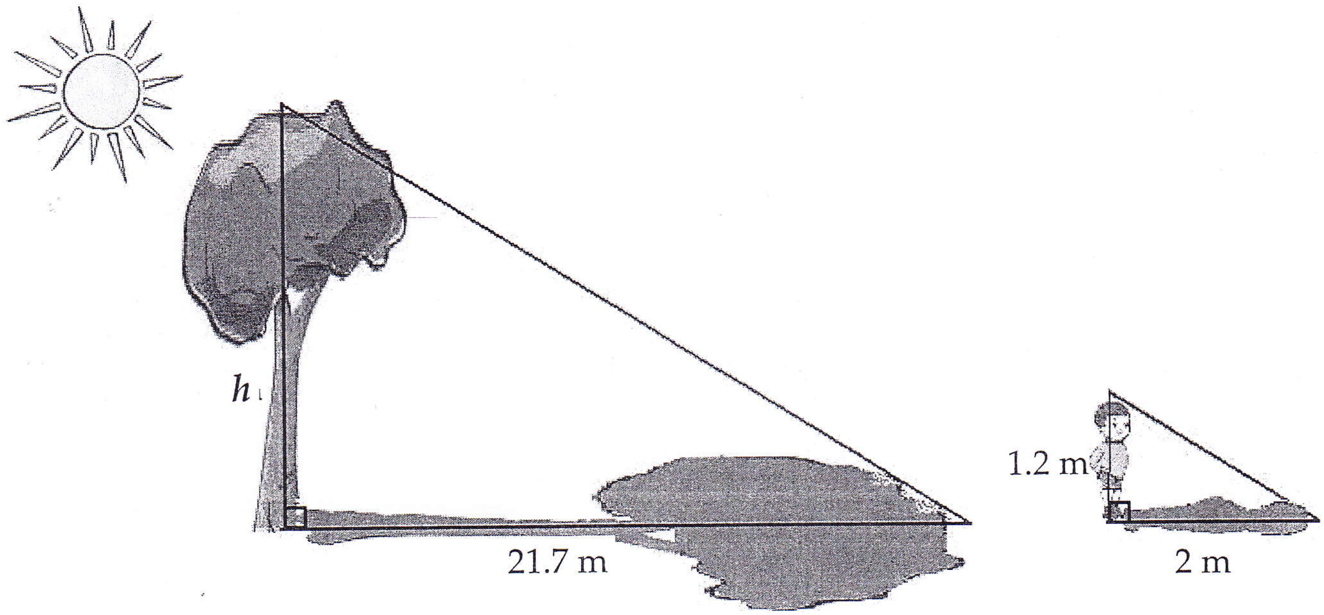
EXAM #4

MFM2P EXAM
June 2012

Solutions

A) Measurement and Trigonometry

A1) A 1.2 m tall boy casts a 2 m long shadow. At the same time of day, a tree has a shadow that is 21.7 m long. **Determine** the height of the tree.



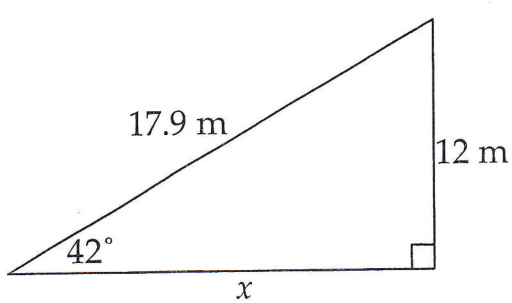
$$\frac{h}{1.2} = \frac{21.7}{2}$$

$$(1.2) \cdot \frac{h}{1.2} = \frac{21.7 \cdot (1.2)}{2}$$

$$h = 13.02$$

$\therefore$ The tree is 13.02m tall

A2) **Determine** the value of x. You may use trigonometry or Pythagorean theorem.



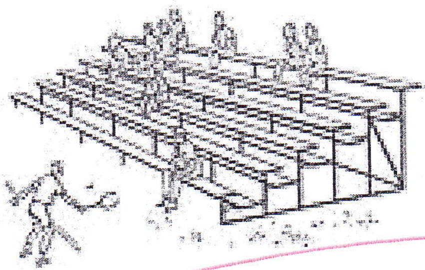
$$a^2 + b^2 = c^2$$
$$x^2 + (12)^2 = (17.9)^2$$
$$x^2 = 320.41 - 144$$
$$x^2 = 176.41$$
$$x = \sqrt{176.41}$$
$$x \cong 13.3$$

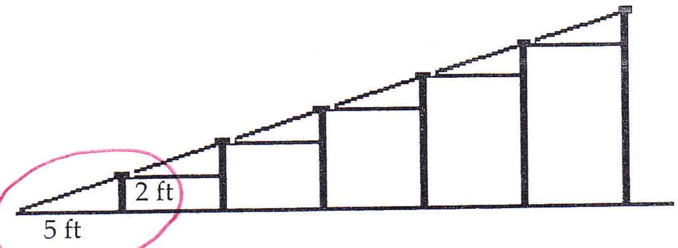
OR

$$\cos\theta = \frac{adj}{hyp}$$
$$\cos 42^\circ = \frac{x}{17.9}$$
$$17.9 \cos 42^\circ = x$$
$$13.3 \cong x$$

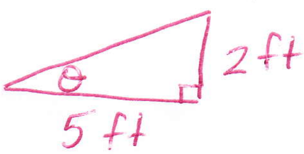
$\therefore x \cong 13.3m$

A3) Portable bleachers have a rise of 2 and run of 5.
The safe angle of inclination for bleachers is between 15° and 30°.





a) **Determine** if this set of bleachers has a safe angle of inclination.
Show your work.



$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{2}{5}$

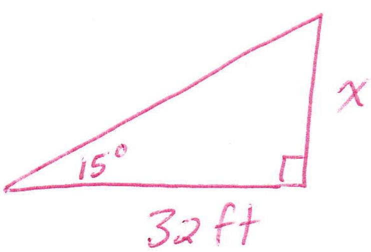
$\theta = \tan^{-1}\left(\frac{2}{5}\right)$

$\theta \cong 22°$

Good! between 15° & 30°

∴ The angle of inclination of the bleachers is safe.

b) A set of bleachers fits in a 32 ft length from front to back.
What do you think would be the best height for the bleachers?
Justify your response.

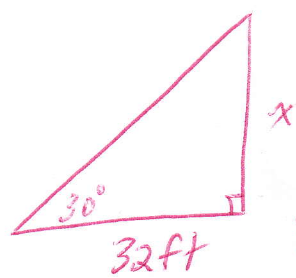


$\tan\theta = \frac{opp}{adj}$

$\tan 15° = \frac{x}{32}$

$32\tan 15° = x$

$8.6 \cong x$



$\tan\theta = \frac{opp}{adj}$

$\tan 30° = \frac{x}{32}$

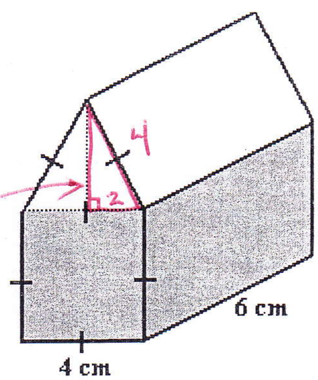
$32\tan 30° = x$

$18.5 \cong x$

∴ The height of the bleacher can be between 8.6m & 18.5m.

A4) A toy company is designing a SOLID wooden block in the shape shown below. The bottom portion is a rectangular prism and the roof is a triangular prism. The triangle is equilateral.

a) **Determine** the total volume of wood needed for the whole block.



Find height

$a^2+b^2=c^2$

$2^2+4^2=c^2$

$4+16=c^2$

$20=c^2$

$\sqrt{20}=c$

$4.5 \approx$

Rectangular Prism

$V=lwh$

$=(4)(6)(4)$

$=96$

Triangular Prism

$V=\frac{1}{2}bhl$

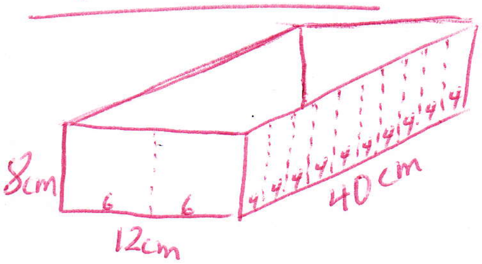
$=\frac{1}{2}(4)(4.5)(6)$

$=54$

∴ The total volume is approx. $150\,cm^3$

b) Maria designs a box that fits 20 blocks. Her box is 40 cm x 12 cm x 8 cm. **Design** another box that would hold 20 blocks, but will use less cardboard.

Maria's Box



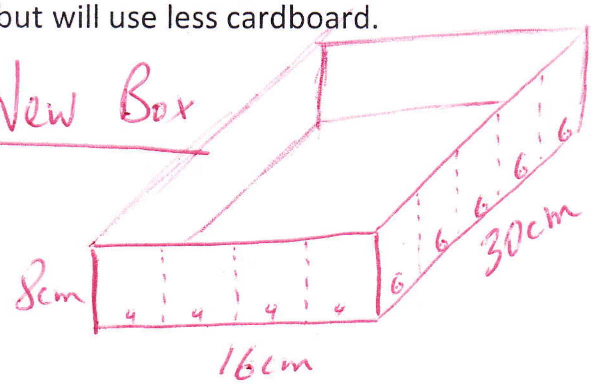
$S.A.=2(8)(12)+2(40)(8)+2(40)(12)$

$=192+640+960$

$=1792$

Maria's Box has a surface area of $1792\,cm^2$

New Box



$S.A.=2(8)(16)+2(30)(8)+2(30)(16)$

$=256+480+960$

$=1696$

∴ The New Box will have a surface area of $1696\,cm^2$

B) Modelling Linear Relations

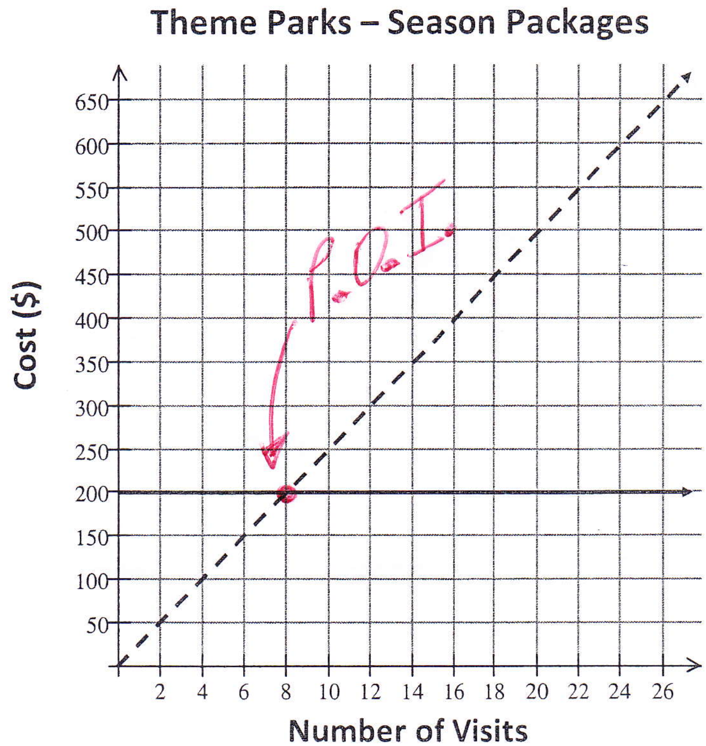
B1) Ahmed is comparing the packages offered by two theme parks.

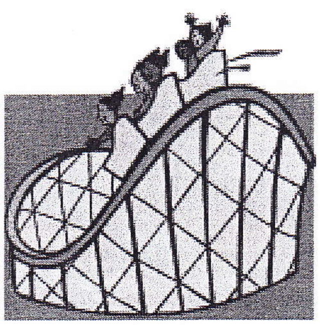
- **Fun Park** charges \$25 per visit
- **Water World** charges \$200 for the entire season

Legend

Fun Park - - - - - - - -

Water World ————————





State the point of intersection of the lines.
Explain its meaning for this situation.

The P.O.I. is at (8, 200)

\# of visits

Cost (\$)

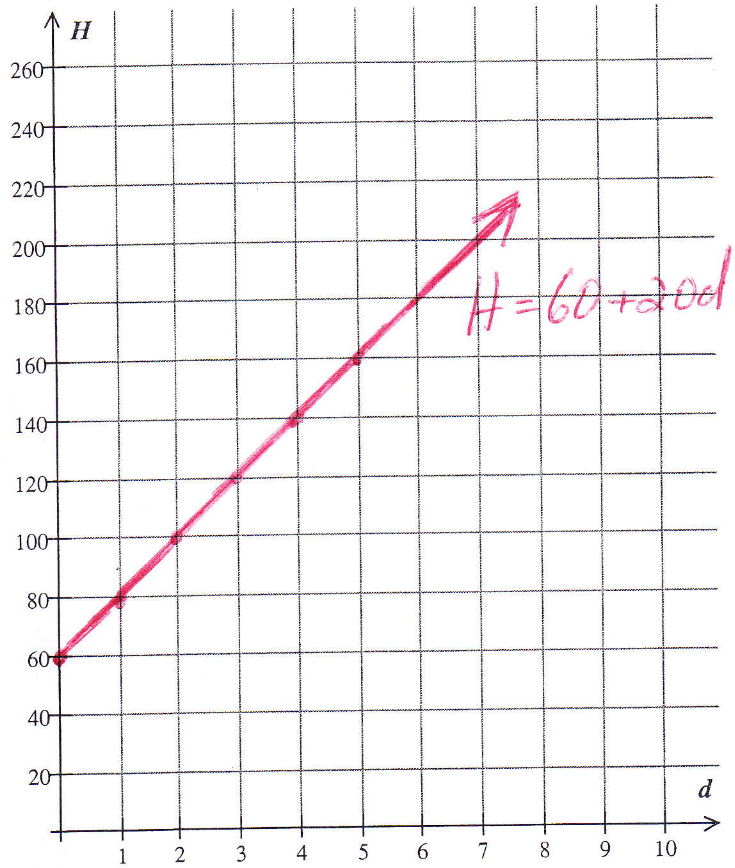
① If you go to a theme park <u>less</u> than 8 times, Fun Park is cheaper

② If you go to a theme park <u>more</u> than 8 times, Water World is cheaper

③ If you go <u>exactly</u> 8 times, they both cost the same, \$200.

B2) The relation $H = 60 + 20d$ models the growth of a sunflower planted in a garden.
H represents the height of the plant in millimetres.
d represents the number of days the plant has been growing.

a) **Graph** the relation.



You could use this table.

d	H
0	60
1	80
2	100
3	120

b) At the end of the season the sunflower was 1100 mm tall.
Determine how many days the flower has been growing.

$H = 60 + 20d$

Sub in $H = 1100$

$1100 = 60 + 20d$

$1100 - 60 = 20d$

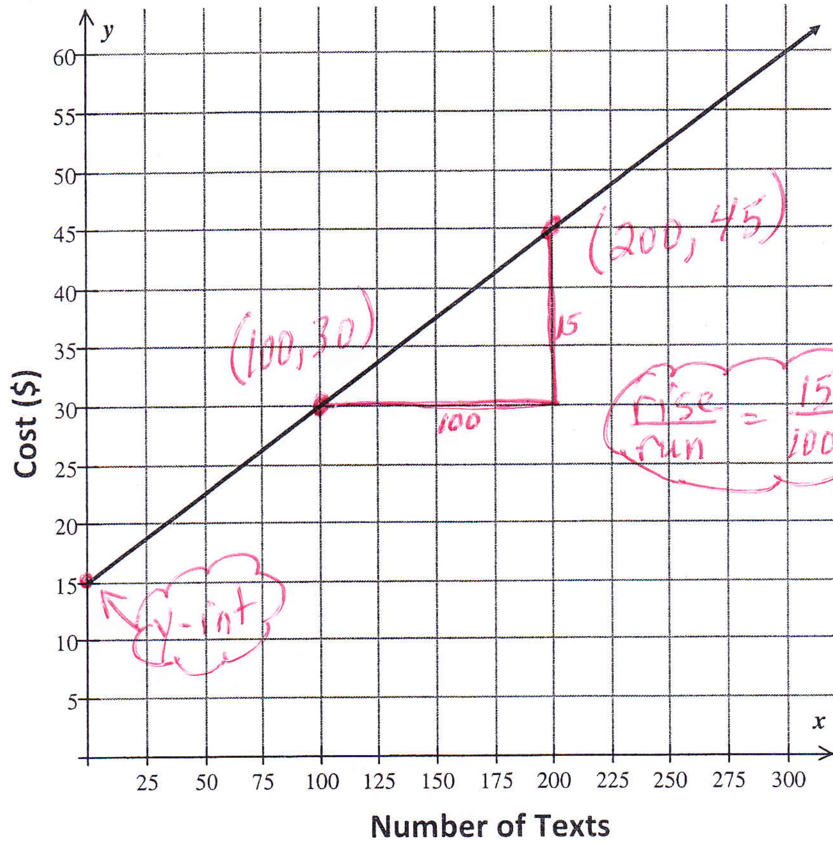
$\frac{1040}{20} = \frac{20d}{20}$

$52 = d$

$\therefore$ The flower has been growing for 52 days

B3) The Yappy Cell Phone Company is offering a new text messaging service. The graph shows the total cost for texting.

Cell Phone Costs



a) **State** the y-intercept.
Explain what it means in this situation.

The y-int is 15.

It represents the initial cost, \$15, that you would pay having made 0 texts.

b) **Determine** the cost per text message.
Show your work.

$$\text{Slope} = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{45-30}{200-100} = \frac{15}{100} = 0.15$$

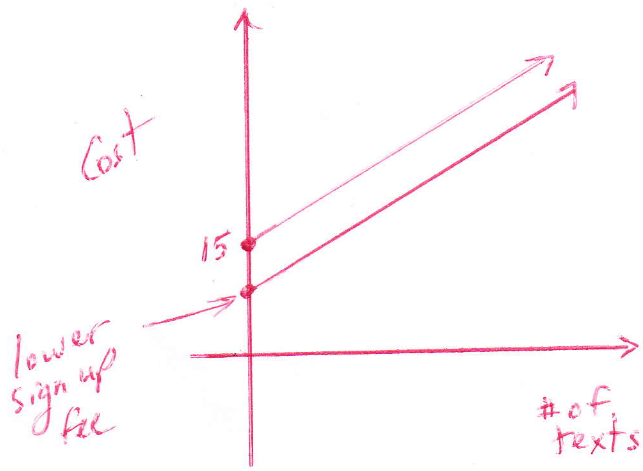
∴ The cost per text is \$0.15

c) **Determine** an equation that models Yappy's charges for texting.

$$C = 0.15n + 15$$

Cost ($) — # of texts

d) Another cell phone company charges the same rate for texting but offers a lower sign-up fee. **Compare** this company's graph to the graph for Yappy.



The graphs would be parallel, but Yappy's line would be higher.

e) Yappy currently charges $30 to send 100 texts. **Describe** <u>two</u> possible ways they could change their fees so that sending 100 texts cost $25.

① Lower their initial fee to $10 instead of $15.

$$C = 0.15n + 10$$

Sub in $n = 100$ to check

$$C = 0.15(100) + 10$$
$$C = 15 + 10$$
$$C = 25$$

② Change their rate to $0.10/text instead of $0.15/text.

Sub in $n = 100$ to check

$$C = 0.10n + 15$$
$$C = 0.10(100) + 15$$
$$C = 10 + 15$$
$$C = 25$$

B4) A baker sells 3 cakes and 1 pie for \$44.50 to Natasha.
He sells 2 cakes and 1 pie to Bilal for \$32.50.

a) **Solve** the system to **determine** the price of 1 cake and the price of 1 pie.

① $3x + y = 44.50$

② $2x + y = 32.50$

Subtract: $x = 12$

Sub in $x = 12$ into ①

$3x + y = 44.50$

$3(12) + y = 44.50$

$36 + y = 44.50$

$y = 44.50 - 36$

$y = 8.5$

∴ The price of 1 cake is \$12 and price of 1 pie is \$8.5

b) Another baker charges more for cakes and less for pies.
Create a system of equations for this new situation.
Explain your answer.

Let a cake cost \$15 and a pie cost \$5

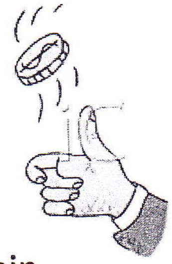
3 cakes + 1 pie will cost
$3(\$15) + 1(\$5) = \$50$

2 cakes + 1 pie will cost
$2(\$15) + 1(\$5) = \$35$

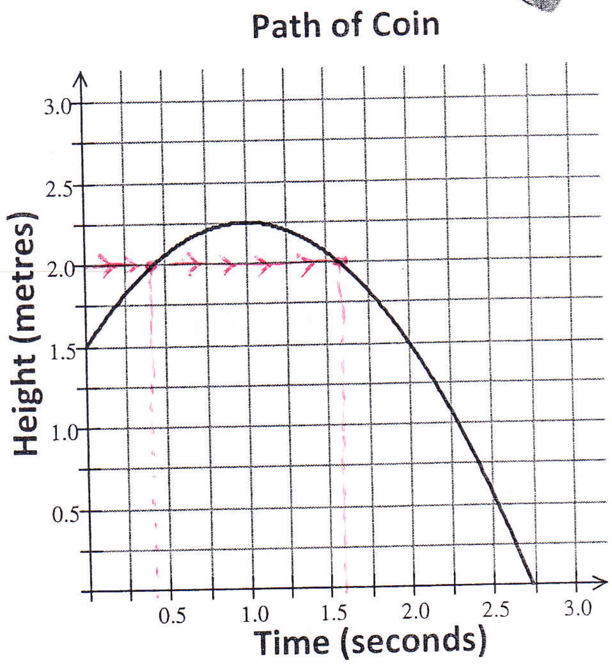
∴ $3x + y = 50$

$2x + y = 35$

C) Quadratic Relations



C1) Jamal flips a coin. The coin's path is modeled by the graph below.



Determine the following:

a) The maximum height of the coin.

Max is 2.25m

b) How long the coin is in the air.

2.75 seconds

c) The time(s) when the coin is 2 metres in the air.

Approx. 0.4 seconds and 1.6 seconds

These grids or algebra tiles could help you to expand and/or factor.

C2) a) Given $y=(x-2)(x+5)$

Expand and **simplify**: $y = x^2 + 3x - 10$

State the y-intercept: -10

	x	-2
x	x^2	$-2x$
5	$5x$	-10

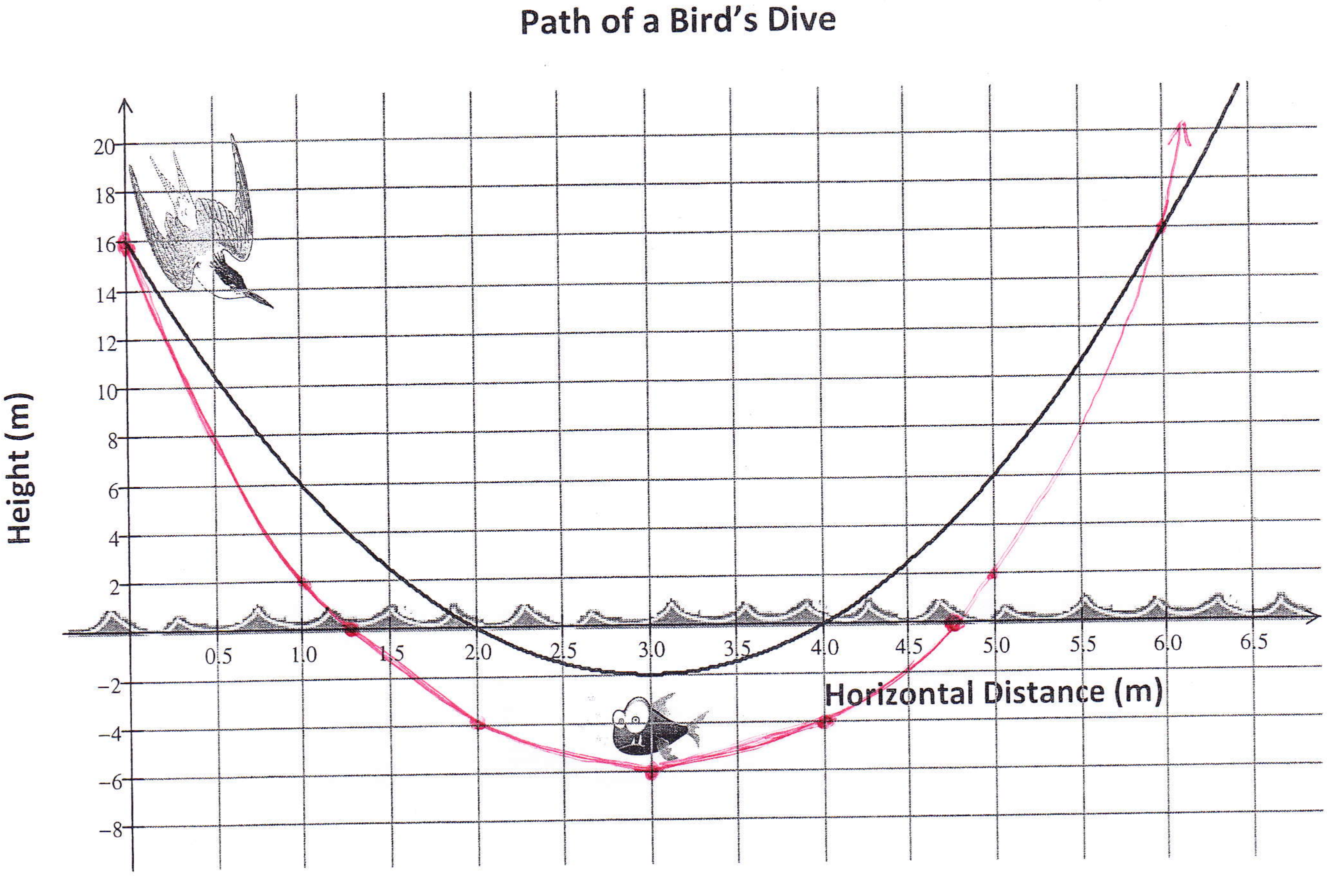
$3x$

b) Given $y=x^2+6x+8$

Factor: $y = (x+4)(x+2)$

State the x-intercepts: $(-4,0)$ & $(-2,0)$

C3) A bird sees a fish under the water and dives down to catch it.



a) **State** the y-intercept.
Explain what it means in this situation.

y-int is 16m
This represents the height of the bird when it begins to dive down.

b) **State** the zeros shown on the graph.
Explain what they mean in this situation.

(2,0) & (4,0)
The zeros represent the distances where the bird enters and exists the water.

c) **State** the vertex.
Explain what it means in this situation.

Vertex is (3, -2)

It means that at a distance of 3m, the bird/fish is 2metres under the water.

d) If the fish was deeper in the water, the path of the bird would change.
Sketch a possible new path of the bird on the original grid.
Describe the important features of the new parabola and how they may have changed.

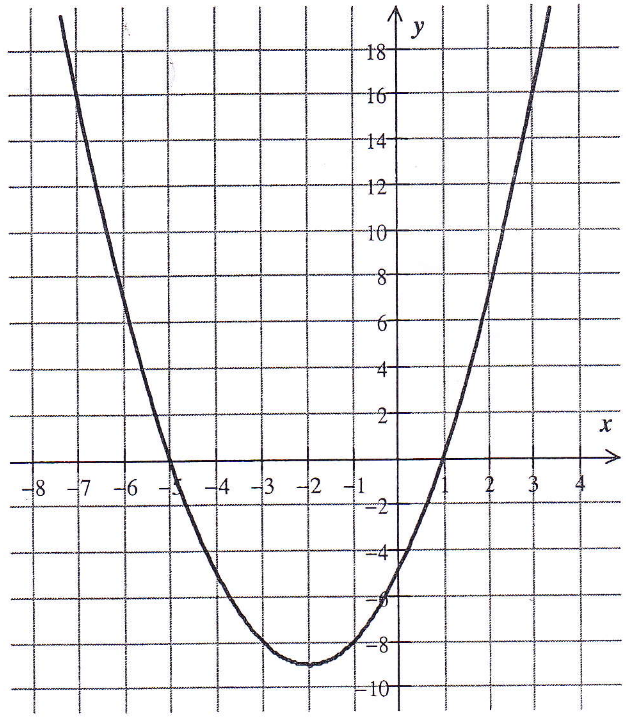
on grid

changed
- Vertex is lower (3, -6)
- the zeros are now at (1.25, 0) & (4.75, 0)

same
- parabola still opens down
- y-int is still at 16 m.

★ Answer will vary ★

4) A teacher gave her class the following problem:



" Model the following relationship in a different form. For example, write as an equation or a table of values. "

Three students gave the following solutions.

<table>
<tr><th>Solutions</th><th>Is the student's solution correct? Explain why or why not.</th></tr>
<tr><td><u>Heather's Solution</u>
<table><tr><th>x</th><th>y</th></tr><tr><td>-5</td><td>0</td></tr><tr><td>-3</td><td>-8</td></tr><tr><td>-1</td><td>-9</td></tr><tr><td>1</td><td>0</td></tr><tr><td>3</td><td>16</td></tr></table>wrong!</td><td>This table has a mistake.
should be (-1, -8)</td></tr>
<tr><td><u>Mike's Solution</u>
$y=(x+5)(x-1)$
correct!</td><td>- Mike's zeros are (-5,0) & (1,0). Good ✓
- $y = (x+5)(x-1)$
$= x^2 - x + 5x - 5$
$= x^2 + 4x - 5$ ← Mike's y-int. Good ✓
- Mike's equation opens up. Good ✓</td></tr>
<tr><td><u>Suresh's Solution</u>
$y=x^2-4x-5$
Wrong!</td><td>- Suresh's y-int is -5. Good ✓
- $y = x^2 - 4x - 5$
$= (x-5)(x+1)$
Suresh's zeros are (5,0) & (-1,0). These are wrong.</td></tr>
</table>